J. Fluid Mech. (1988), *vol.* 192, *pp.* 175–192
Printed in Great Britain

Experimental characterization of steady two-dimensional vortex couples

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(Received 8 April 1987 and in revised form 13 November 1987)

We study the evolution of unsteady two-dimensional vorticity structures surrounded by fluid at rest. The flow is initiated by a short fluid impulse in a horizontal layer of mercury and is constrained to be two-dimensional by a vertical uniform magnetic field. The impulse is generated by an electric pulse between two electrodes, and a flow circulation can be produced by diverting part of the current through the external frame. The velocity field is measured from the streaks of small particles floating on the free upper surface, and the vorticity is then obtained by means of an analytical interpolation and differentiation. The flow always evolves toward a set of independent steady structures with symmetry which are either circular vortices (monopoles) or couples (dipoles). The latter have a linear or circular steady motion depending on the flow circulation around them. The region of non-zero vorticity is always close to a circle. The steadiness is confirmed by plotting the vorticity versus the stream function in the frame of reference moving with the couple. We obtain a curve, as appropriate for a steady solution of the Euler equation. The slope of this curve is either constant or has no maximum. We suggest that this result could correspond to a general stability condition. The interaction between two symmetric couples at various angles of incidence yields two new couples by exchange of their vortices. Oscillations of the resulting couples are often damped by releasing a circular vortex.

1. Introduction

Isolated couples† of two-dimensional counter-rotating vortices with a fast translating motion have been known as steady solutions of the Euler equation since the 19th century. They have the remarkable property of transporting momentum and mass like a solid body does. The relevance of such couples to real fluid flows was clearly demonstrated in the two-dimensional wake behind a cylinder by Couder (1984). They can even dramatically dominate the whole wake when the cylinder oscillates (Couder & Basdevant 1986). These experiments were performed in a flat soap film, and similar results were obtained in a layer of mercury where three-dimensional perturbations are inhibited by a magnetic field (Papaliou 1985). Two-dimensional vortex couples were also observed in an ordinary wake behind a long cylinder by Williamson & Roshko (1986), but strong oscillations of the cylinder seem to be necessary for their observation. Vortex couples have motivated several works in geophysical fluid dynamics and are often called modons in this context. When they interact with turbulence, vortex couples are generally less stable than the circular vortices that are clearly observed in the oceans (McWilliams 1985) and in numerical

† We call a couple a set of two counter-rotating vortices to make the distinction with a set of two corotating vortices called a pair, following a suggestion of Couder & Basdevant (1986).

computations of two-dimensional turbulence (McWilliams 1984). They could nevertheless have a mixing role because of their ability to transport matter efficiently over long distances. Furthermore there is some hope for a theory of two-dimensional complex flows that considers simple vortex structures as elementary building blocks, like solitons for gravity waves (Flierl, Stern & Whitehead 1983). This idea was also conceived for drift wave turbulence in plasma physics (Makino, Kamimura & Taniuti 1981), which can be approximated by the same equations as quasi-geostrophic flows.

Different analytical and numerical steady translating solutions of the two-dimensional Euler equations are described in the literature. The simplest one is a set of two point vortices, which can be considered as an extremely loose couple. By contrast a very compact couple is described by Lamb (1945) (also by Batchelor 1967) and has some analogy with the spherical Hill vortex: the vorticity is proportional to the stream function inside a circle and vanishes outside. A similar solution was found for an asymmetric couple moving steadily on a circle by Flierl *et al.* (1983). A family of couples made of two areas of uniform vorticity separated by an arbitrarily small gap has also been studied by Deem & Zabusky (1978) and Pierrehumbert (1980). Finally, more complex couples have been obtained numerically by McWilliams & Zabusky (1982) and McWilliams (1983). These steady translating solutions are found to be stable to small (two-dimensional) perturbations in numerical computations, but the theory of their stability remains to be done. Different authors have proposed that the stable structures emerging from a complex unsteady flow with enstrophy (vorticity squared) dissipation should have the minimum total enstrophy compatible with a given value of some conserved quantities, like energy and momentum. Leith (1984) has calculated the corresponding vorticity distribution for circular vortices, but no similar calculation has been done for couples.

We have studied the structure of vortex couples emerging from a complex unsteady flow. The experiments are performed in a horizontal layer of mercury submitted to a uniform vertical magnetic field. A short jet is created by the electromagnetic force due to a pulse of electric current between two electrodes. The three-dimensional perturbations are quickly damped (in a fraction of a second) by the eddy currents and the flow becomes independent of the vertical coordinate, outside a thin Hartmann boundary layer at the bottom. This complex unsteady two-dimensional flow then evolves more slowly and generates a steady vortex couple with possibly one or two circular isolated vortices. Similar experiments were performed by Flierl *et al.* (1983) by injecting a short water jet in a rotating tank, where three-dimensional structures are suppressed by the Coriolis force. In both cases the two-dimensional regime is not affected by the external force except for a fairly weak linear friction, due to the effect of viscosity in the bottom boundary later. However the dynamics of the initial three-dimensional stage is different, and so is the resulting two-dimensional unsteady flow. Moreover, we are able to measure the vorticity field with a reasonable precision ($\approx 10\%$) from flow visualization by a method described in §3. General constraints due to the conserved quantities are discussed in §4.1. Couples with a given momentum but no global rotation are obtained when the whole current is flowing from one electrode to the other, and these are studied in §4.2. Flows with non-zero circulation are described in §4.3 when part of the current returns through the outer frame. Finally, interactions between two vortex couples are described in §5.

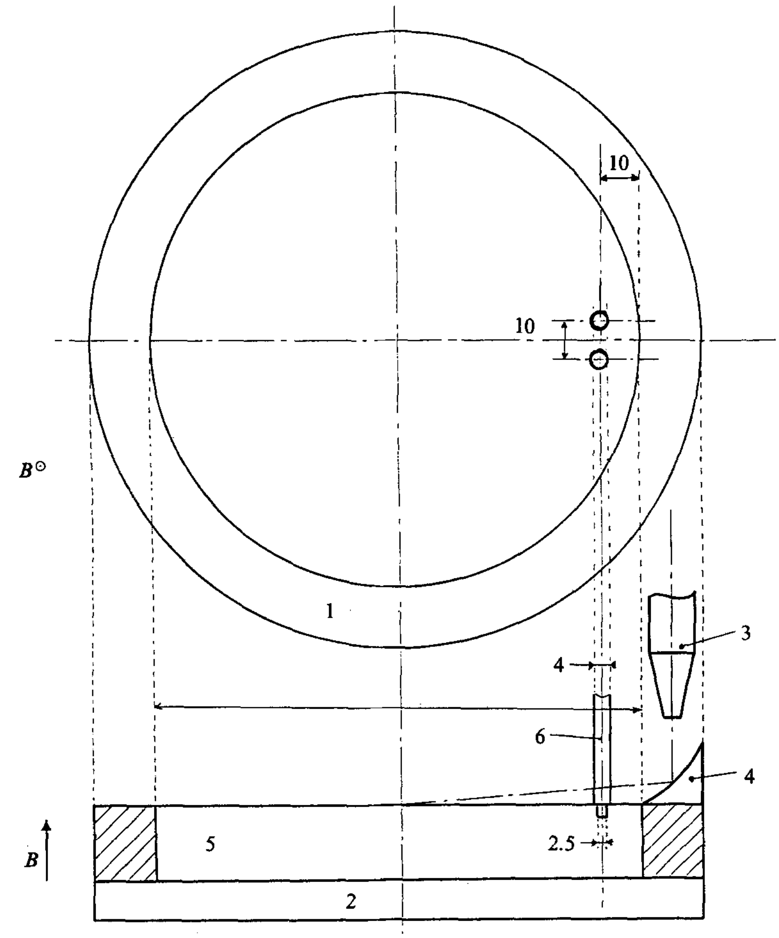


FIGURE 1. Sketch of the apparatus: 1, copper frame; 2, insulating bottom; 3, optical fibres for the illumination; 4, cylindrical mirror; 5, mercury; 6, electrodes. Dimensions are in mm.

2. Experimental apparatus

The apparatus is a circular box 16 cm in diameter, containing a horizontal layer of mercury, 2 cm in depth. The choice of this depth is the result of an optimization: the three-dimensional perturbations are better suppressed when the layer is shallower, but the Hartmann friction in the bottom boundary layer then has more influence (Sommeria & Moreau 1982). The box is located inside a long vertical solenoid producing a steady vertical magnetic field $B = 0.38$ T. The spatial uniformity of this field is better than 1%. The box is made of an electrically insulating bottom (in ertalon) and a copper ring, chemically coated with nickel for protection against mercury. The upper surface is free (in contact with pure argon to avoid oxidation) but its vertical deformation is negligible. The typical velocity is of the order of 10 cm/s, which corresponds to a Reynolds number of 10^4. Pulses of electrical current are introduced through two fixed copper electrodes (figure 1). A second identical pair of electrodes can be located anywhere around the box in order to simultaneously produce a second vortex couple and study the interaction with the other one at different angles of incidence.

All flow measurements are made from the photographic streaks of small silica particles (50 μm in diameter) floating on the free upper surface. For each run a sequence of photos is taken at intervals of 0.5 s with a time of exposure of 0.1 s. The light is carried by two beams of optic fibres and spread in nearly horizontal sheets by two cylindrical mirrors (figure 1). Seen from above, the particles then appear bright on a black background. The meniscus of the mercury surface must be concave to avoid the falling of the particles on the edge. This concave meniscus is obtained by coating the lateral walls with a thin layer of electrolytic gold. This layer is quickly dissolved but the underlying nickel is then wetted by mercury. An excellent electrical contact is also obtained by this technique and the same coating is used for the electrodes.

3. Velocity and vorticity measurements

The flow is measured from the length and orientation of short particle streaks (approximated as straight segments), using a method described by Nguyen Duc & Sommeria (1985). The velocity field is obtained by the tedious process of pin-pointing the two ends of each streak on a large digitizing table connected to a computer. The image is obtained by a direct projection of the negative film about 1 m in size and the precision of the process is about 2 % of the maximum velocity. The sense of each vector is chosen by continuity, and by looking at the global motion of the flow structure. The next step is to interpolate the velocity field between the points of measurement in order to compute the vorticity and to get the velocity vectors on a square lattice. Particular care is brought to the interpolation method to optimize the use of our laboriously acquired data. Different computer programs were tested by generating velocity vectors of the analytically defined vortex couple of Lamb (1945) at random locations and comparing the interpolation with the original field. A method called 'spline thin shell' developed by Paihua Montes (1978) appeared to be especially good. The interpolating function for each velocity component is an analytical expression, the coefficients of which are calculated in order to minimize the second-order derivatives. The interesting features of this method are that it (i) avoids spurious oscillations (unlike polynomial interpolations), since the spline interpolation is a function with minimum second derivatives; (ii) is completely independent of the axes x and y that we choose; and (iii) gives the exact measured value at the measurement points wherever they are located (the method can be modified to account for a given uncertainty of the measurements but we did not use this option).

The two interpolated velocity components v_1 and v_2 at any point $M(x, y)$ are calculated by the relations

$$v_j(M) = a_j + b_j x + c_j y + \Sigma\, d_j(i)\, MM_i^2 \ln MM_i^2 \quad (j = 1, 2), \tag{1}$$

where MM_i is the distance between M and one of the N points of measurement M_i. The first three terms correspond to a least-square linear fit for each velocity component. The coefficients $d_j(i)$ are calculated by solving the linear system of N equations obtained by applying (1) to the N measurement points. Equation (1) can be analytically differentiated to yield the vertical component of vorticity ω and the divergence at any point:

$$\omega = \frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}, \tag{2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y}. \tag{3}$$

The measured divergence field is weak and appears to be independent of the flow structure, which means that it results mostly from experimental errors. Since divergence and vorticity are computed by a similar process, the typical errors on these two fields should be the same. Therefore the typical error on the vorticity must be of the same order as the measured divergence. To get an estimation of the relative error on the vorticity, we normalize the divergence measured at each point of the calculation grid by a typical magnitude of the vorticity. This typical magnitude is defined as half the difference between the maximum and minimum vorticity in the vortex couple $\frac{1}{2}(\omega_{max} - \omega_{min})$. The values of this normalized divergence, obtained at each grid point, are partitioned in bins, and the probability of occurrence in each bin is plotted in a histogram (figures 4*f* and 8*f*). We see on these figures that for 95% of the points the measured divergence is smaller than $0.1\frac{1}{2}(\omega_{max} - \omega_{min})$, so that the relative error on the vorticity can be estimated as less then 0.1 for 95% of the grid points. We should notice that the best accuracy is obtained when the measurement points are evenly scattered. Indeed, when two streaks are very close, a small error in each streak measurement can lead to large relative error in the difference, hence a large error in the vorticity. Beside the small errors of measurements, a few mistakes are sometimes made (for instance the wrong direction for a velocity vector). These mistakes are immediately detected by the strong local extremum in a plot of the divergence isovalues, and are then corrected.

The stream function ψ is computed from the vorticity field by a numerical integration of the Poisson equation

$$\nabla^2 \psi = -\omega. \tag{4}$$

The domain of integration is a rectangle which extends well outside the region of vorticity to avoid edge effects. The boundary condition on this rectangle is $\psi = 0$. A grid of 30×30 points is found to be sufficient for this computation. The stream function in a frame of reference translating or rotating with the couple can be then calculated by a simple transformation:

translation $\qquad \psi \rightarrow \psi - U_x y + U_y x,$

rotation $\qquad \psi \rightarrow \psi + \Omega OM^2,$

where U_x and U_y are the two components of the translation velocity, and Ω the rotation rate around a centre O.

4. Generation of a vortex couple

4.1. *General comments*

A flow impulse is generated by a current pulse of intensity I between 6 and 10 A and duration $\tau = 0.3$ s, sufficiently short to get a local flow surrounded by fluid at rest. A dip of the free surface with small-scale structures is then observed between the two electrodes during less than one second, revealing a strong three-dimensional motion.

However, these perturbations disappear quickly and a much smoother and slower two-dimensional flow replaces it. This process should occur in a time of the order of $\rho/\sigma B^2(a/L)^2$ for a turbulent structure of horizontal scale L, according to Sommeria & Moreau (1982) (a denotes the thickness of the mercury layer, ρ the density of the fluid, and σ its conductivity). This estimation yields for example a time 1 s for a scale L of 5 mm, so that most of the kinetic energy should be indeed in the two-dimensional modes of motion after one second. The only effect of the magnetic field is then a friction force, due to the Hartmann boundary layer on the bottom, proportional to v/t_{H} where $t_{\mathrm{H}} = a/B(\rho/\sigma\nu)^{\frac{1}{2}} = 16.6$ s. This is much larger than a turnover time (typically less than 1 s) and the only dynamical effect of this friction is an overall decay of all the quantities related to the velocity field (illustrated on figure 7). We did not check the two-dimensionality by direct measurements but all our results agree well with an interpretation using the two-dimensional Navier–Stokes equations, as did some previous experiments obtained with different forcing in a similar kind of apparatus (Sommeria 1986; Verron & Sommeria 1987).

The two-dimensional flows in unbounded fluid at rest at infinity are constrained in the limit of vanishing viscosity by the conservation of the following integral quantities (see Batchelor 1967):

$$\Gamma = \iint \omega \,\mathrm{d}A,$$

$$\boldsymbol{P} = \iint \boldsymbol{r}\omega \,\mathrm{d}A,$$

$$M = \iint r^2\omega \,\mathrm{d}A,$$

$$E = \iint \psi\omega \,\mathrm{d}A.$$

The enstrophy $V = \iint \omega^2 \,\mathrm{d}A$ is also conserved in the absence of viscosity. However, in a two-dimensional turbulent flow, the enstrophy is transferred to small scales by a cascade process, and is dissipated by viscosity, even in the limit of vanishing viscosity. When $\Gamma = 0$, (as in §4.2) $\boldsymbol{P}, M, E$ are equal respectively to the momentum, angular momentum and energy of the flow (divided by $a\rho$). This interpretation of the quantity $\boldsymbol{P}$ must be clarified when we take into account the effect of the boundaries, supposed far from the vorticity region. Indeed, since the centre of mass of the fluid as a whole does not move, the total momentum is actually always zero. The momentum of the localized flow is exactly balanced by the momentum of a weak irrotational flow necessary to satisfy the boundary conditions of zero normal velocity. This irrotational flow can be considered as due to fictitious image vortices outside the box and should have a negligible influence on the dynamics when the couple is far from the walls. When $\Gamma \neq 0$ (as in §4.3) the angular momentum and energy tend to infinity with the box diameter and the physical meaning of $\boldsymbol{P}, M$ is different. $\boldsymbol{P}/\Gamma$ indicates the position of the 'centre of vorticity' and M/Γ is a measure of the spatial extension of the vorticity distribution. The conservation of these two quantities means that the vorticity distribution cannot move away from its region of production. This constraint is illustrated by the circular motion of the vortex couples with non-zero circulation.

The current density $\boldsymbol{j}$ associated to the electric pulse produces locally a force per unit of volume $\boldsymbol{j} \times \boldsymbol{B}$ during the time τ. By integrating this force in the fluid volume, we can calculate the total momentum initially generated, $P = IBd\tau/\rho a$, perpendicular to the line joining the two electrodes (d is the distance separating the two electrodes). The actual momentum of the jet is in fact typically five times smaller, and can make a small angle with this direction. Therefore an important part of the initial momentum is dissipated in the strongly perturbed three-dimensional initial stage.

When a part cI of the current pulse returns through the frame, a non-zero circulation exists at large distance, proportional to the non-zero current flux. By integrating the driving force $\boldsymbol{j} \times \boldsymbol{B}$ on a vertical cylinder surrounding the two electrodes, we can indeed relate the total circulation produced around the vorticity region to the flux of current density cI, $\Gamma = cIB\tau/\rho a$. The actual circulation measured in the two-dimensional regime is typically four times smaller, so there is also a strong decrease of the circulation in the initial stage of motion.

The velocity scale U and size L of an isolated vortex couple without circulation can be expressed as a function of E and P:

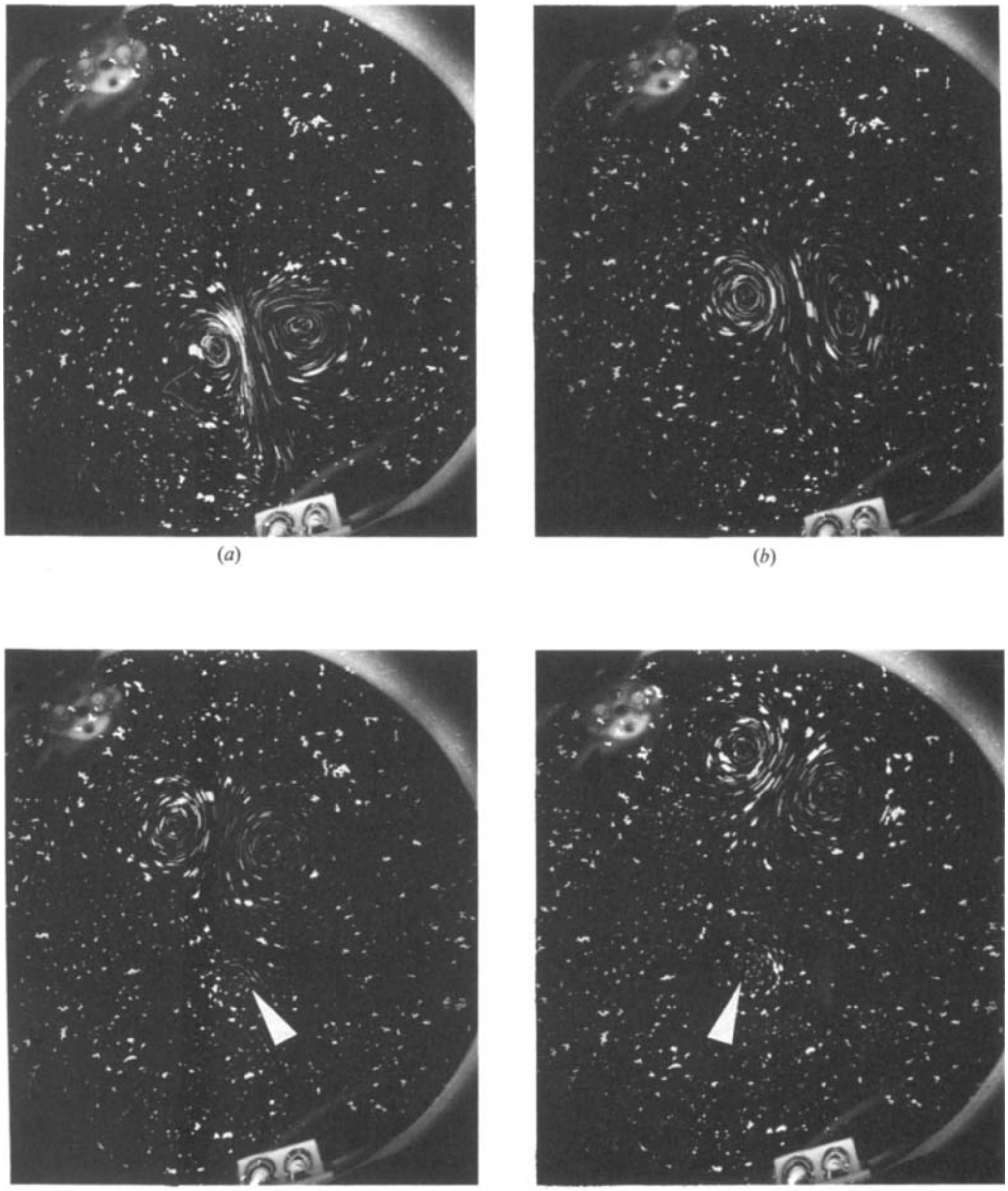
$$U = E/P, \quad L = P/E^{\frac{1}{2}}.$$

The direction of translation is determined by the direction of $\boldsymbol{P}$, and the position of the trajectory by M/P. At that point any structure and shape is compatible with any initial value of these conserved quantities. However, the enstrophy of the steady couple must be smaller than the enstrophy of the initial two-dimensional state. In the general case with non-zero circulation the structure is constrained by the 'parameter of asymmetry', which can be defined as $g = \Gamma^2/E$.

4.2. *Couples without circulation*

We report in this section the results obtained when no current is passing through the frame, so that the initial circulation due to the forcing vanishes. The flow can behave in different ways, depending on the turbulent fluctuations in the initial three-dimensional stage. However, in all cases the system clearly evolves toward a set of isolated steady vorticity structures. The formation of a symmetric translating couple without circulation is the most common situation. This couple has a line of symmetry along the direction of translation and another in the perpendicular direction. Its formation is often associated with the release of one or two circular vortices (figure 2). The direction of translation is generally roughly perpendicular to the line joining the two electrodes but exceptions exist. The couple sometimes has a small circulation, so that it is slightly asymmetric in the transverse direction and moves on a circle of large radius, but we shall leave these cases for the next section.

An easy way to check the steadiness is to plot the position of the two vortex cores at successive times, as in figure 3. After a transitory stage of a few seconds, the distance between the two cores is constant within 5% until the couple arrives close to the external frame, where it increases because of the influence of the image vortices. At a very close distance two new vortices are produced by a boundary-layer detachment from the wall, which gives the impression that the images 'come out of the wall'. A similar process was observed for single vortices by Sommeria (1988). As a consequence of these effects, a complex flow, localized near the wall, is produced and no couple is reflected: in other words the interaction with the wall is completely inelastic. Far enough from the external frame, the mean motion is a straight steady translation, but fluctuations are not quite damped before the couple reaches the lateral wall. Similar oscillations were observed in numerical computations



(a) (b) (c) (d)

FIGURE 2. The evolution toward a symmetric couple. The photos are taken at times (*a*) $t = 1$ s, (*b*) 2 s, (*c*) 5 s, (*d*) 7 s after the current pulse. The circular isolated vortex is shown by the arrow. Time of exposure 0.1 s.

(McWilliams 1983). The effect of Hartmann friction is also observed but has only a small influence during the time it takes for the couple to cross the box.

The properties of the couples change from one run to another, even when the initial current pulse is identically reproduced. The flow evolution is determined in the strongly unstable initial stage of motion. The distance between the two vortex centres in the laboratory frame of reference is typically between 1.5 and 2.5 cm and

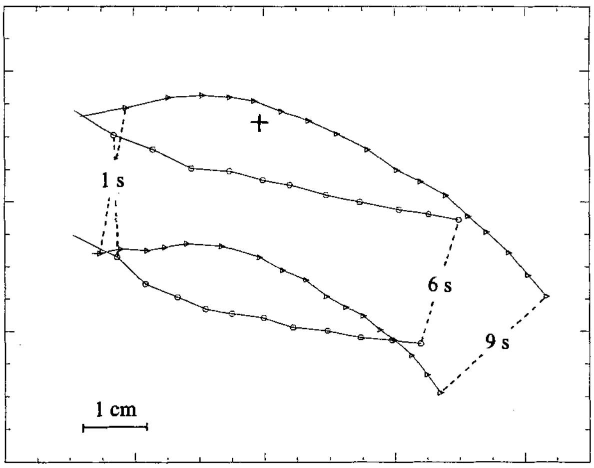


FIGURE 3. Successive positions at intervals of 0.5 s of the two vortex centres for couples without circulation; the elapsed times from the electric pulse to the first and last displayed positions are indicated; the cross corresponds to the box centre; △, couple of figure 2; ○, couple analysed in figure 4.

the speed of translation is between 1 and 2 cm/s. The ratio of the velocity at the middle of the couple in the laboratory frame of reference to the speed of translation is a characteristic of the shape and vorticity structure. This ratio can have any value between 2.8 and 4, revealing that different shapes and structures are possible for the couple. The small values correspond to a couple somewhat elongated in the direction of translation while the large ones correspond to an elongation in the perpendicular direction. However, because of the limited precision for the measurement of this ratio (5%), we cannot exclude the possibility of a discrete set of stable states instead of a continuous one.

The measured velocity field of a symmetric couple is represented in the laboratory frame of reference in figure 4(*a*). The arrows are limited to the vortex region, where an appreciable velocity can be measured. Outside this region, a few scattered points with zero velocity are introduced in the digitizing process, but cannot be seen on figure 4(*a*). The result of the interpolation on a periodic grid is represented on figure 4(*b*). The streamlines in the reference frame translating with the couple are shown in figure 4(*c*), and the shape of the couple can be defined by the boundary between the open streamlines and the closed ones. The corresponding vorticity field is shown in figure 4(*d*). Each point of measurement is represented in figure 4(*e*) using its stream function ψ and vorticity ω as coordinates. The points collapse approximately on a curve with two branches, one of which is on the ψ-axis and corresponds to the irrotational flow outside the couple. This behaviour corresponds to what is expected for the steady solutions of the Euler equations. This result is thus a direct proof that we are close to a stationary state and provides an experimental determination of the structure function $\omega = f(\psi)$, which is linear in this example. The linearity is not uncommon but other relations are obtained as well with other examples and are discussed in the next section. The dispersion of the points is somewhat larger than the experimental errors inferred from the divergence histogram (figure 4*f*), which is due to the existence of the oscillations.

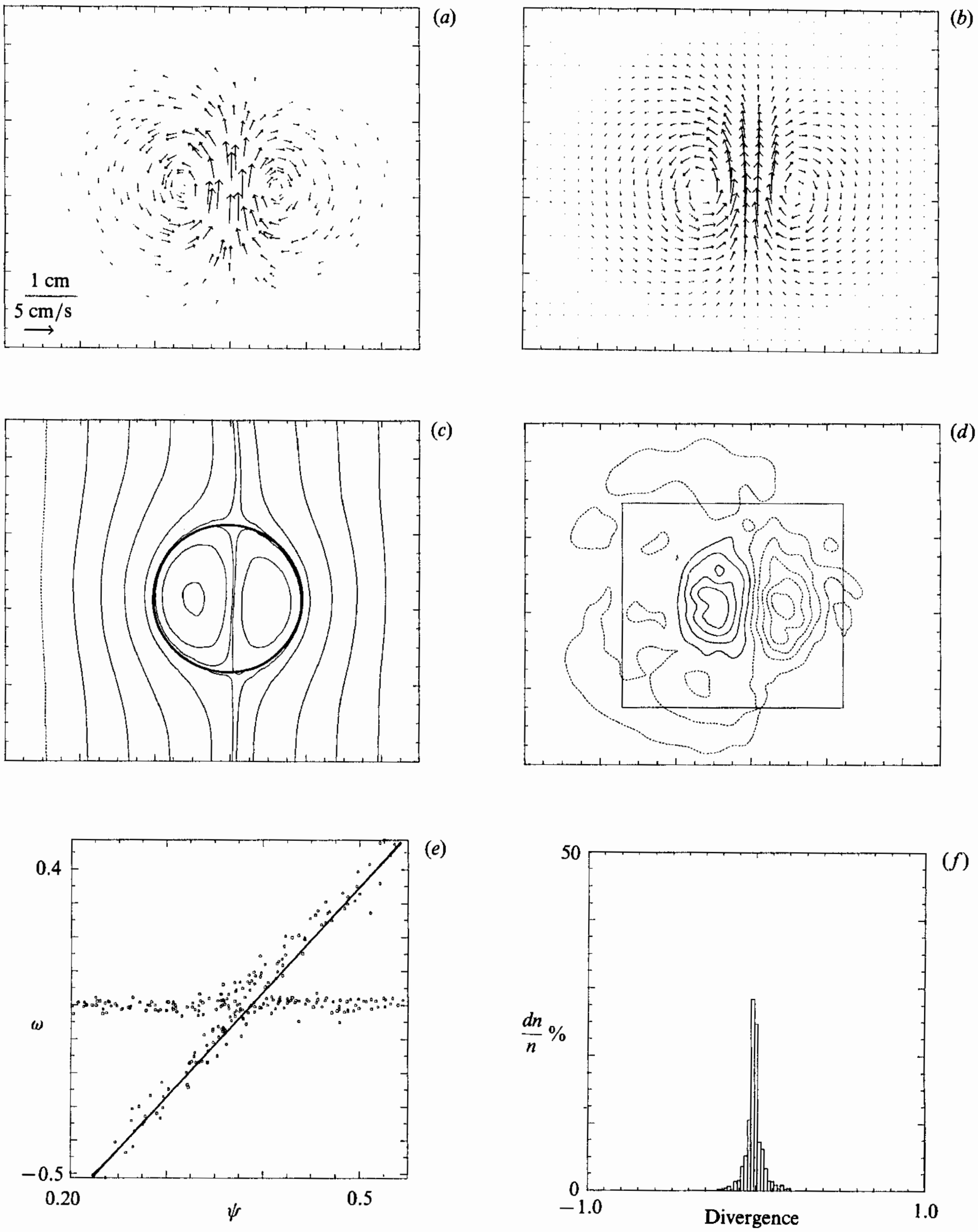


FIGURE 4. Numerical processing of a couple with a negligible circulation (factor of asymmetry $g = 0.022$): (*a*) velocity vectors corresponding to the streaks; (*b*) interpolated velocity; (*c*) streamlines in the frame of reference translating with the couple (at speed 1.41 cm/s), the boundary of the closed streamlines is indicated; (*d*) iso-vorticity lines (interval 2.6 s^{-1}, the dashed lines correspond to negative values); (*e*) representation with (ω, ψ)-coordinates in the translating frame of reference (inside the square in *d*); (*f*) histogram of the divergence (normalized by $\frac{1}{2}(\omega_{max} - \omega_{min})$).

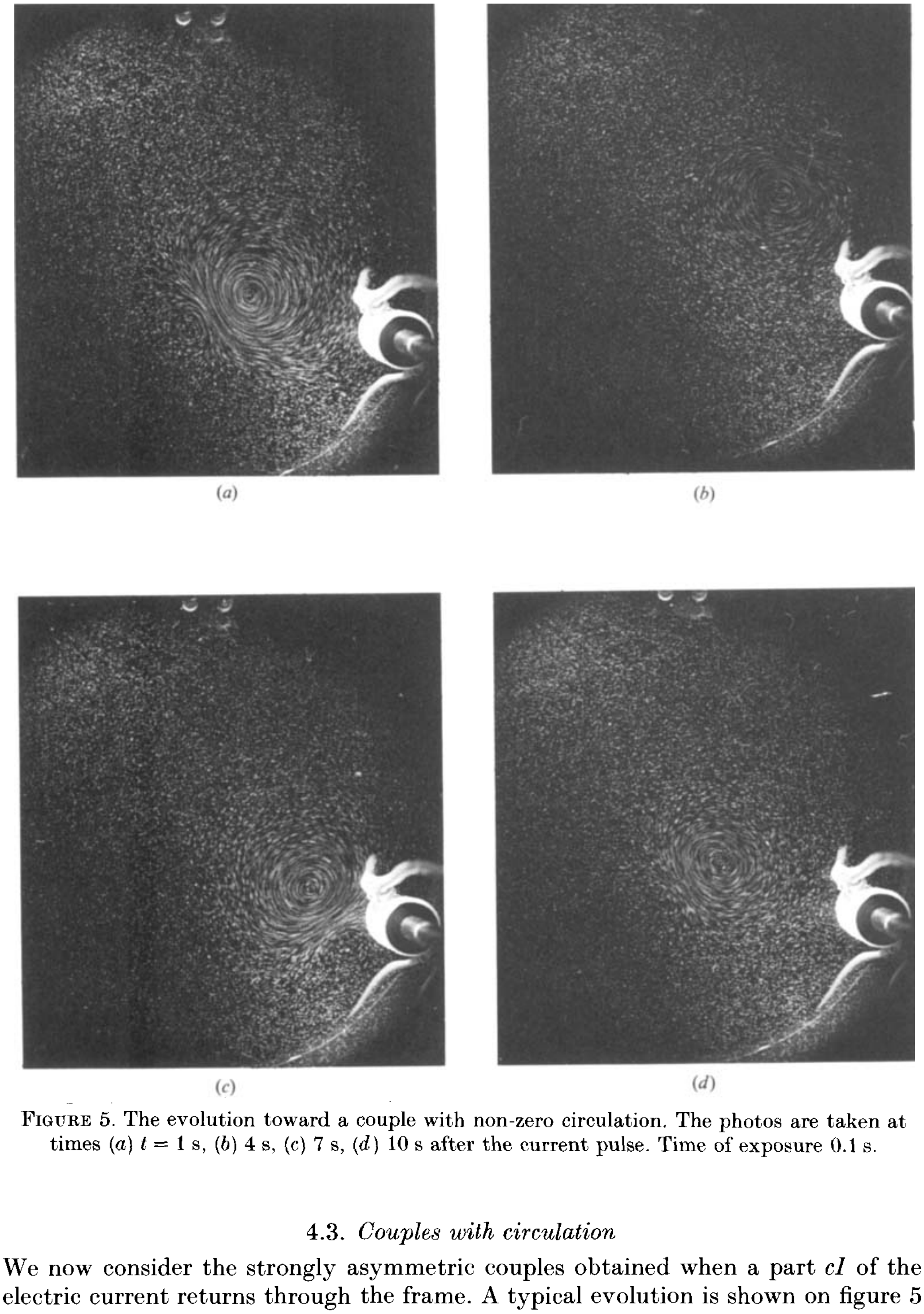


FIGURE 5. The evolution toward a couple with non-zero circulation. The photos are taken at times (*a*) $t = 1$ s, (*b*) 4 s, (*c*) 7 s, (*d*) 10 s after the current pulse. Time of exposure 0.1 s.

4.3. *Couples with circulation*

We now consider the strongly asymmetric couples obtained when a part cI of the electric current returns through the frame. A typical evolution is shown on figure 5 and examples of trajectories are plotted on figure 6. These couples reach a steady state more quickly than the symmetric ones, probably because of the vorticity mixing due to their rotation, and no circular vortex is released. Furthermore the couple can be followed for much longer times because the circular trajectory never

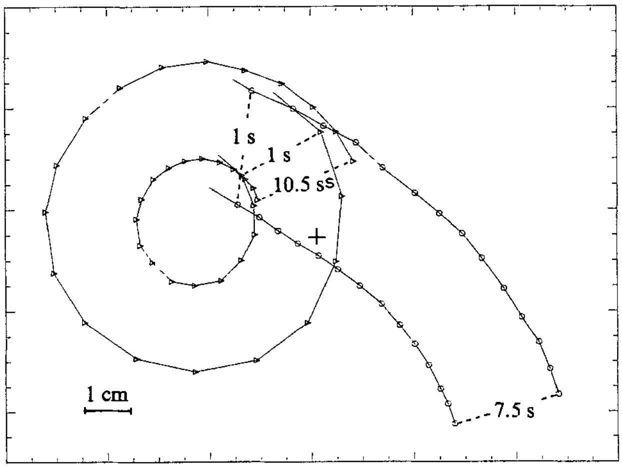


FIGURE 6. Successive positions at intervals of 0.5 s of the two vortex centres for couples with a circular motion. Same indications as in figure 3. △, corresponds to figure 5; ○, is analysed in figure 9(*b*).

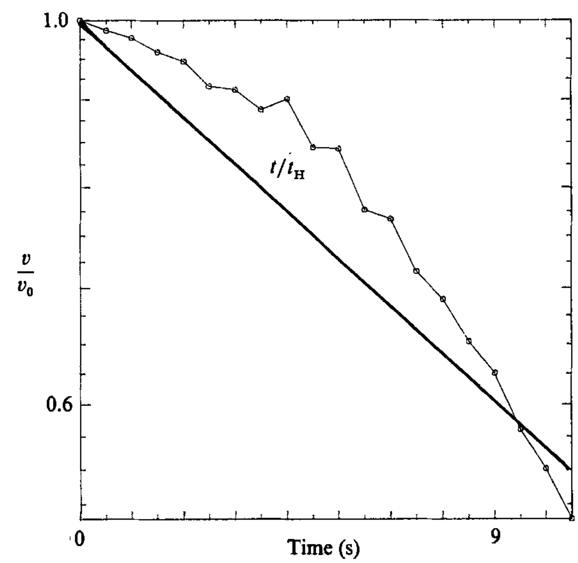


FIGURE 7. Evolution of the angular velocity of the couple motion showing the Hartmann decay (couple of figure 5).

reaches the external frame. The motion is slowly damped by the Hartmann friction as shown in figure 7. The mean dissipation rate corresponds well to the laminar value t_H^{-1}, as could be expected from Sommeria (1988), but the decay is in fact slower at the beginning and faster later. This discrepancy can be explained by the influence of the image vortices behind the lateral walls. Notice that the decay does not produce an inward spiralling motion as in the results of Couder & Basdevant (1986). This latter feature must be due to a different law of dissipation that modifies the shape of the couple.

While the couples can be strongly asymmetric in the transverse direction, a good symmetry is always observed about the line joining the two vortex cores. An example of velocity, stream function and vorticity field is shown on figure 8. The

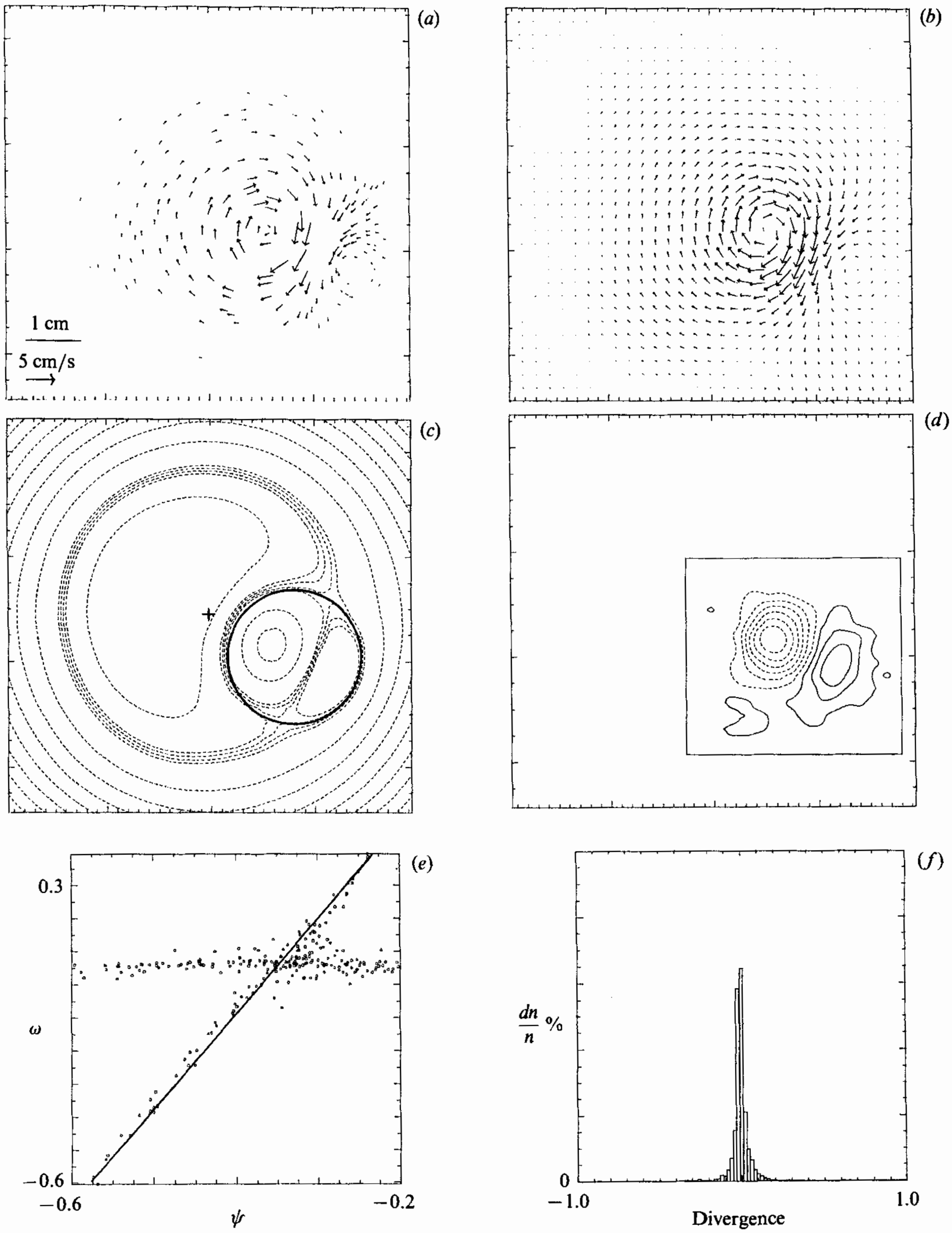


FIGURE 8. Analysis of a couple with non-zero circulation (the one of figure 5, $g = 1.02$) in the same way as in figure 4. (a) velocity field; (b) interpolated velocity; (c) streamlines in a frame of reference rotating with the couple around the centre, indicated by the cross, at angular velocity 0.68 rad/s; (d) isovorticity lines at interval 2.32 s^{-1}; (e) representation with (ω, ψ)-coordinates; (f) histogram of the divergence normalized by $\frac{1}{2}(\omega_{max} - \omega_{min})$.

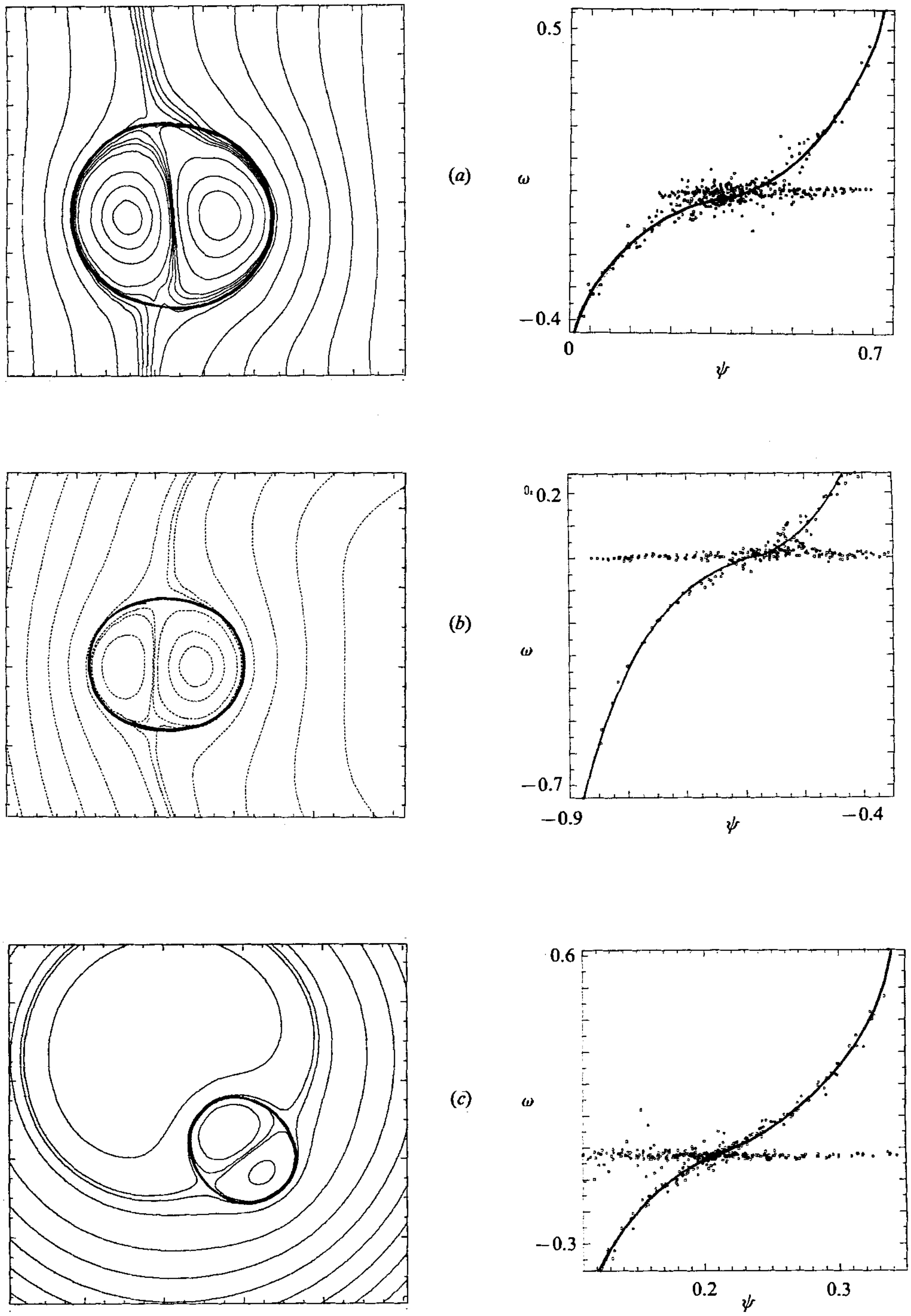


FIGURE 9. Streamlines in the frame of reference where the couple is stationary (left) and relation ψ, ω (right) for three examples with a nonlinear function f. (*a*) $c = 0$, $g = 0.09$ (couple of figure 2), isoline interval $0.05\ \mathrm{cm^2\,s^{-1}}$; (*b*) $c = 0.11$, $g = 0.19$, isoline interval $0.125\ \mathrm{cm^2\,s^{-1}}$; (*c*) $c = 0.2$, $g = 0.566$, isoline interval $0.17\ \mathrm{cm^2\,s^{-1}}$.

relation between ω and ψ is defined more precisely than for the couples without circulation, which confirms that the oscillations are weaker. The relation between the stream function and vorticity is linear in this case but other functions can also be obtained, as for couples with zero circulation. The shape of this particular couple is circular within the measurement errors, so that the flow corresponds to the analytical solution of Flierl *et al.* (1983). The circle represented in figure 8(*c*) has a radius b such that $kb = 3.83$ ($J_1(kb) = 0$), where $\omega = k\psi$, corresponding to the boundary of the couple in the analytical solution. The experimental boundary fits well with a circle of such radius. The parameter $e = b/A$, where A is the radius of the trajectory of the circle centre, is equal to 0.66 in this example. However other values of this parameter can be obtained, depending on the run.

Examples of shape and nonlinear vorticity structures are given in figure 9 for different asymmetry factors. Different relations $\omega = f(\psi)$ can be obtained. The asymmetry factor g increases as the proportion c of the current diverted to the lateral walls is increased. Nevertheless, there is a variability of the flow structure due to instabilities in the strongly perturbed initial stage of motion, as for couples with zero circulation.

In spite of this variability of the function $f(\psi)$, we always find that the derivative f' is maximum at the singular points corresponding to the two vortex cores, but has no local maximum elsewhere (but has a minimum for $\omega = 0$). This property is also observed for the couples obtained in the numerical experiments of McWilliams (1983), and in recent numerical simulations of forced two-dimensional turbulence by Legras, Santangelo & Benzi (1988). It seems therefore to be a quite general feature of vortex couples emerging in a complex two-dimensional flow. We cannot determine whether this property is a consequence of the initial conditions, or whether it corresponds to a general stability criterion. The second alternative could provide an explanation for the common occurrence of couples with a linear function f. Indeed a couple with a maximum of f' would be unstable and evolve until a state or marginal stability is reached, corresponding to a constant f'. However we cannot test this hypothesis experimentally since we cannot control the initial vorticity distribution. According to this hypothetical criterion, couples formed by two patches of uniform vorticity, with slightly smoothed boundaries would be unstable, since the function f' has then a strong maximum at the edge of the patches. The stability of couples with patches of uniform vorticity has been shown in the numerical simulations of Overman & Zabusky (1982), but the boundaries of the patches correspond to a discontinuity for the vorticity and the function f. Therefore this result is not in contradiction with our hypothesis. A theorem stating that a steady inviscid two-dimensional flow is always stable if f' is negative everywhere was derived by Arnol'd (1965), but it applies only in a confined domain, so that there is also no contradiction with our hypothesis for isolated vorticity structures.

5. Interaction between vortex couples

The vortex couples can be considered as particles whose interactions we want to study. For that purpose we generate simultaneously two vortex couples at two sets of electrodes (figure 1) with an identical current pulse of 9 A lasting 0.3 s. We select the runs for which the two couples have a straight translating motion and collide near the centre of the box. The size and speed of the two couples is then similar (but not precisely equal). The angle of incidence is chosen (by moving one electrode couple around the box) to be either 90°, 120° or 180°. The collision is also characterized by

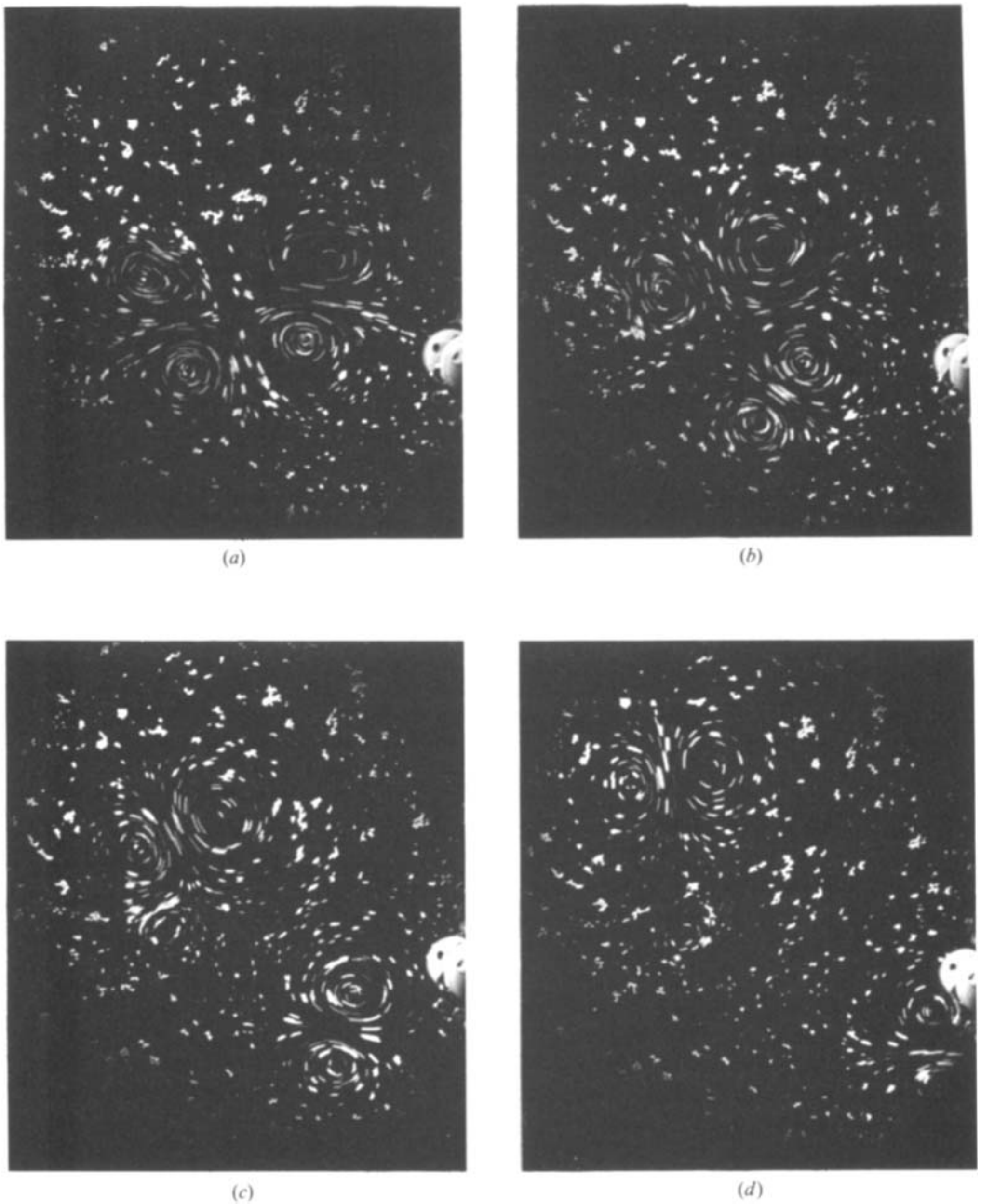


FIGURE 10. An interaction between two translating couples at an angle of incidence 120°. The successive photos are taken at the times (*a*) $t = 1$ s, (*b*) 2.5 s, (*c*) 4 s, (*d*) 5.5 s after the electric pulses. Time of exposure 0.1 s.

the impact parameter (distance normal to the trajectory at the interaction point) but this quantity cannot be controlled because of variations of the trajectory directions due to the initial turbulence.

In spite of these random differences between the various runs some features are quite reproducible. The interaction produces two new couples resulting from the exchange of the vortices as seen on figure 10. For head-on collisions, the two new couples are very similar to the initial ones but turned through 90°. For an angle of incidence of 120° the couple produced on the side of the small angle is compact while the second one is loose. This effect is still more important for a smaller angle of incidence (90°). A similar behaviour was also obtained by Couder & Basdevant (1986) in numerical computations and experiments with soap film. We observe again the tendency of the couples to eliminate the fluctuations and reach a symmetric state by releasing a circular vortex. The variation of energy and enstrophy during the interaction corresponding to figure 10 is less than the typical experimental error ($\approx 10\%$). The enstrophy dissipation during this interaction seems then to be small, but the precision and the number of cases that we were able to process are too limited to allow a general conclusion.

6. Conclusions

We have first confirmed by new examples that our experimental method makes possible the quantitative investigation of two-dimensional flows at high Reynolds numbers ($\approx 10^4$). Furthermore we are able to measure the vorticity field from particle streaks with a precision better than 10% of its maximum value. We should emphasize the importance of the interpolation process for this achievement, as we learned from several attempts with different standard methods.

We find that a two-dimensional vorticity distribution surrounded by fluid at rest always evolves in a few turnover times towards a set of independent steady structures, which are couples or single circular vortices. This result is obtained both from the evolution of a fluid impulse, and from the intermediate state produced during the encounter of two couples. It seems thus to be quite general. The steady state is actually obtained for couples with circulation, but oscillations around a steady state are present in couples without circulation. A line of symmetry joining the two vortex centres is always obtained. Couples without circulation are also symmetric about the direction of translation. It seems that each structure attains the highest symmetry compatible with the conserved quantities.

The shape of the couples (i.e. the boundary of the vorticity region) is always close to a circle but can be elongated either in the direction of translation or in the perpendicular one. The function $\omega = f(\psi)$ which characterizes the steady state is either linear or is such that the derivative f' always has singular maxima at the vortex cores, but no local maximum elsewhere. Further study is needed to decide whether this quite general property is related to a stability criterion, or is just a consequence of the nature of the initial conditions.

The initial purpose of these experiments was to create an hypothetical kind of two-dimensional turbulence, considered as a 'gas' of vortex couples with infrequent interactions. This goal was not possible to achieve because of the absence of reflection of the couples on the external frame. We have observed that the interaction between two compact couples generates two new couples but one of them is sometimes very loose and could probably be easily broken by another interaction. However, vortex couples also have a tendency to be produced from any isolated vorticity structure, so it is not clear whether a dilute gas of vortex couples would be stable even without the destruction near the walls.

The authors have benefited from useful discussions with Y. Couder, S. A. Maslowe and K. Moffatt. The numerical code for the spline interpolations was kindly given by D. Creutin.

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